Using MEM to describe CDMA Multiple Access Interference

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Abstract: The Maximum Entropy Method (MEM) is used to evaluate and characterize the Multiple Access Interference (MAI) of a direct sequence Code Division Multiple Access (CDMA) system. The MAI distribution is derived and compared with the general assumption that the MAI random variable tends to a Gaussian random variable (this assumption is commonly known as the Gaussian Assumption). The influence of the number of users, K, the spreading sequence length, N, and fading on the MAI are investigated. Further, by using the Minimum Relative Entropy Method (MREM), which is related to the MEM, the discrimination information (relative entropy) between the MAI distribution and an equivalent Gaussian distribution is examined. To investigate the influence of fading on the MAI, a Nakagami-m fading channel model is considered. By altering the parameter m of the Nakagami-m distribution, the degree of fading can be varied, from no fading to severe fading. As input to MEM power moments are computed to calculate the MAI distribution.

I. INTRODUCTION

The importance of CDMA as multiple access technology for mobile communications has been confirmed with the recent ETSI decision to adopt Wideband CDMA (WCDMA) and Time-Division CDMA (TD-CDMA) as basis for the implementation of the Universal Mobile Telecommunication Service (UMTS). Unlike traditional FDMA and TDMA communication systems where the system capacity is primarily bandwidth limited, CDMA system capacity is interference limited. It is therefore very important to accurately model the interference of such a system and to derive accurate capacity estimates, design optimal error control and spreading codes and to apply efficient interference cancellation techniques. One of the most common approximations of the MAI is to consider only second order moment information (the variance) of the MAI random variable [1]. Unfortunately large errors in the estimate of the interference can result from casual and gratuitous appeal to this approximation, especially when the transmission channel is faded [2]. The reasoning behind the Gaussian Assumption (GA) is that the overall MAI consists of the contributions from many different interferers and the interference from each interferer consists of the contributions from many different chips of the spreading sequence. In such situations, central limit arguments may apply and the overall MAI may be approximated by a Gaussian random variable. By examining the decision statistics (see Section II), it is clear that the contributions from different interferers are dependent, and the contributions from different chips of the same interferer are also dependent. Another way of proving this statement is to look at the Error Free Run (EFR)[1] distribution of the received signal. Simulated results of the EFR distribution reveal that especially for a small number of users, errors tend to be bursty, indicating a dependence among the errored bits. Since the channel errors are dependent, the widely used Central Limit Theorem for independent random variables does not apply. There is no proof that the MAI converges to a Gaussian random variable, not even under stationary unfaded conditions. It is therefore reasonable to assume that the exact conditions for the overall MAI to converge to a Gaussian random variable are not clear, making the application of the GA confusing and invalid. It has also been shown that the GA give very poor estimates of the MAI random variable [3] under certain conditions. With this in mind we set out to predict the MAI in a more accurate way using the Maximum Entropy Method (MEM) and the related Minimum Relative Entropy Method (MREM).

In a previous paper [4] we have shown that MEM [5], [6] and MREM [7] give very accurate, reliable and numerical stable results when used to evaluate

[1] An EFR distribution gives the probability that a certain number of consecutive errors will occur between correctly received bits

average error probability in digital communication systems. We extend the results of [4] to investigate the MAI. Using MEM closed form expressions of the MAI distribution can be obtained, making the use of approximations unnecessary. The advantage of having the MAI in a closed form is that it can be used to calculate the CDMA performance with any linear or non-linear modulation scheme. The results presented in this paper is valid, in general, for any linear modulation scheme.

We also try to answer the question "how accurate is the Gaussian Assumption?" using the MREM, instead of the usual comparison of average error rates obtained with the GA and comparing the results to bounds or some other exact formulation. Since the MREM method is based on the general principles of information theory, we can say that the accuracy of the GA is quantified from an information theoretic point of view. By using MREM the accuracy of the MAI distribution relative to a Gaussian distribution with equal second order moments (variance) is quantified. This is accomplished by calculating the missing information between the two distributions. For example when the two distributions are exactly the same, the missing information between the two distributions are zero. As the two distributions differ, the missing information increases.

To investigate the MAI distribution under fading conditions, the Nakagami-m distribution [8], [9], [10] (for brevity called the Nakagami distribution hence forth) is a viable fading model that fits certain urban data better than Rayleigh, Rician or log-normal distributions. The parameter $1/2 < m < \infty$ indicates severest to least fading respectively. The Rayleigh distribution is a special case of the Nakagami distribution when $m = l$. As m increases the distribution tends to an impulse, constituting no fading. It is therefore meaningful to investigate the MAI under Nakagami fading conditions due to its realistic fading statistics and flexibility in adjusting the amount of fading. Using MEM, MREM and the Nakagami fading channel model the MAI, as a function of m, is investigated.

It should be pointed out that this paper presents an alternative way to look at the MAI caused by CDMA communications. Other well known contributions in this area, such as [11], considers simplified models to calculate CDMA system performance, while this paper gives a better mathematical description and intuitive feel for the MAI.

The paper is organized in the following way. In the next section an appropriate model is discussed to investigate the MAI under fading conditions. Section III summarizes the MEM and the MREM formalisms, while Section IV presents numerical examples to illustrate the results. Finally Section V concludes the paper.

II. MULTIPLE ACCESS INTERFERENCE MODEL

To investigate the MAI using MEM it is necessary to derive an expression for the power moments of the MAI random variable which is used as a boundary condition for the MEM. For our derivation a rudimentary modulation scheme such as Coherent BPSK is assumed, but the results and conclusions will hold in general for any linear modulation scheme. An expression for the average error rate is also derived to show that the MEM produces accurate results for CDMA systems.

A. Multiple Access Interference Moments

In our CDMA system there are K users and the data signal, $b_k(t)$, of user k is a sequence of unit amplitude positive and negative rectangular pulses of duration T, described by

$$b_k(t) = \sum_{j=-\infty}^{\infty} b_j^k P_T(t - jT), \tag{1}$$

where

$$P_T(t) = \begin{cases} 1 & \forall \quad 0 \geq T < 1 \\ 0 & \quad \text{otherwise} \end{cases} \tag{2}$$

The data signal of each user is spread by a spreading sequence, given by

$$a_k(t) = \sum_{j=-\infty}^{\infty} a_j^k P_{T_c}(t - jT) \tag{3}$$

with a period of $N = T/T_c$.

The transmitted and received signals can respectively be written as

$$s_k(t) = Aa_k(t)b_k(t)\cos(\omega_c t + \theta_k)\,\forall\, k = 1, 2, \cdots, K \tag{4}$$

and

$$\begin{aligned} r(t) \quad &= \quad A\sum_{k=1}^{K}[\beta_k a_k(t-\tau_k)b_k(t-\tau_k)\cdot \\ &\quad \cos(\omega_c t - \omega_c\tau_k + \theta_k)] + n(t), \end{aligned} \tag{5}$$

where n(t) is white Gaussian noise with double sided spectral density of $N_0/2$, τ_k the random delay of the kth user and β_k is the Nakagami distributed random path gain of the kth user.

We arbitrarily choose user $K = 1$ as the reference user. Since coherent demodulation is assumed, the receiver coherently recovers the carrier phase and delay lock to the desired signal. Therefore, after correlation and demodulation, a signal sample at the receiver low-pass filter can be expressed as

$$\zeta = \int_0^T r(t)a_1(t)\cos\omega_c t dt. \tag{6}$$

By the assumption that the phase delay locking of the receiver to the desired received signal, (6) can be expressed as

$$\zeta = \beta_1 \frac{AT}{2} b_0^1 + \frac{A}{2}\sum_{l=2}^{L} \beta_l \left\{ b_{-1}^l R_{l,1}(t_l) + b_0^l \hat{R}_{l,1}(t_l) \right\} \cos\theta_l + \eta, \quad (7)$$

where b_0^1 represents the information bit detected and b_{-1}^1, is the preceding bit, which due to the channel delay spread, affects the detection of b_0^1 received on the first path between the desired transmitter and receiver.

Due to the asynchronous arrival of the interfering users' codes, (7) contains partial correlations of the regenerated sequence, $a_1(t)$ and a delayed version of the interfering codes defined by

$$R_{k,1}(\tau_k) = \int_0^{\tau_k} a_k(t-\tau_k)a_1(t)dt \quad (8)$$

and

$$\hat{R}_{k,1}(\tau_k) = \int_{\tau_k}^{T} a_k(t-\tau_k)a_1(t)dt. \quad (9)$$

From (7), (8) and (9) it is apparent that the decision statistics of the MAI are indeed dependent. The dependence is between the information bit being detected, b_0^1, and the preceding bit, b_{-1}^1. The question of course is how much does this dependence influence the validity of an approximation to the Central Limit Theorem.

The standard notation of Pursley [12] can be used to evaluate the correlation functions of (8) and (9). This is accomplished by assuming rectangular chips and noting that, as shown by Pursley, for any $0 \geq nT_c \geq \tau \geq (n+1)T_c \geq T$

$$\begin{aligned} R_{k,1}(\tau) &= A_{n_{k,1}}T_c + B_{n_{k,1}}(\tau - nT_c) \quad (10) \\ \hat{R}_{k,1}(\tau) &= \hat{A}_{n_{k,1}}T_c + \hat{B}_{n_{k,1}}(\tau - nT_c) \end{aligned}$$

where

$$\begin{aligned} A_{n_{k,1}} &= C_{k,1}(n-N) \\ B_{n_{k,1}} &= C_{k,1}(n+1-N) - C_{k,1}(n-N) \\ \hat{A}_{n_{k,1}} &= C_{k,1}(n) \\ \hat{B}_{n_{k,1}} &= C_{k,1}(n+1) - C_{k,1}(n) \end{aligned} \quad (11)$$

which is valid for all $k = 1, 2, \ldots, K$. The discrete aperiodic cross-correlation term $C_{k,1}(\cdot)$ is related to the chip sequences a_j^k and a_j^1 via

$$C_{k,1}(n) = \begin{cases} \sum_{j=0}^{N-1-n} a_j^k a_{j+n}^1 & 0 \leq n \leq N-1 \\ \sum_{j=0}^{N-1+n} a_j^1 a_{j-n}^k & 1-N \leq n \leq 0 \\ 0 & |n| \geq N. \end{cases} \quad (12)$$

By defining

$$\alpha = \frac{\beta_k}{T}\left\{ b_{-1}^k R_{k,1}(\tau_k) + b_0^k \hat{R}_{k,1}(\tau_k) \right\} \cos\Theta_k. \quad (13)$$

we have an expression for the MAI random variable. We would like to determine the statistics of α and compare it to a Gaussian random variable. To achieve this we have to calculate the moments of α and apply it to MEM. By following the procedure outlined by Kavehrad [13] it is possible to calculate the power moments of α as expressed in (13) for a Nakagami faded channel.

We will proceed by calculating N_m moments of the random variable α, which is a function of τ_k, Θ_k and b_1^k. Furthermore, α is symmetrically distributed; hence the odd moments of α_k are all zero. Therefore, having the even moments of α_k, one can determine the moments of

$$\alpha = \sum_{k=2}^{K} \alpha_k, \quad (14)$$

using a three step method prescribed in [14], where from the cumulants of the random variables, the moments are obtained.

Since

$$\alpha_k = \frac{V_k}{T}\left\{ b_{-1}^k R_{k,1}(\tau_k) + b_0^k \hat{R}_{k,1}(\tau_k) \right\} \cos\Theta_k, \quad (15)$$

then,

$$\begin{aligned} N_m &= E\{\alpha_k^{2j}\} \quad (16) \\ &= \frac{1}{T^{2j}} E\left\{ \left[b_{-1}^k R_{k,1}(\tau_k) + b_0^k \hat{R}_{k,1}(\tau_k) \right]^{2j} \right\} \cdot \\ &\quad E\left\{ [\cos\Theta_k]^{2j} \right\} E\left\{ V_k^{2j} \right\}, \end{aligned}$$

where j indicates the number of moments.

Since Θ is an independent random variable, it can be dealt with separately. That is, we first evaluate

$$E\left\{ [\cos\Theta_k]^{2j} \right\} = \frac{\binom{2j}{j}}{4^j} \quad (17)$$

and then find the first expectation of (16) as

$$H = \frac{1}{T}\int_0^T \left[b_{-1}^k R_{k,1}(\tau_k) + b_0^k \hat{R}_{k,1}(\tau_k) \right]^{2j} d\tau_k, \quad (18)$$

with the expectation in H over the random delay τ_k.

The expectation of (16) can now be expressed as

$$\begin{aligned} N_m &= E\{z_k^{2j}\} = \quad (19) \\ & \frac{\binom{2j}{j}}{4^j} \frac{E\{V_k^{2j}\}}{N^{2j+1}} \sum_{n=0}^{N-1}\sum_{r=0}^{j} \binom{2j}{2r} \Gamma_{j,r,n} \end{aligned}$$

with $\Gamma_{j,r,n}$ given by [13]

$$\Gamma_{j,r,n} = \sum_{i=0}^{2r} (-1)^i \frac{(B_{n_{k,1}})^i}{(\hat{B}_{n_{k,1}})^{i+1}(i+1)} \cdot \frac{\binom{2r}{i}}{\binom{2(j-r)+i+1}{i+1}}. \quad (20)$$

$$\Big\{(A_{n_{k,1}}+B_{n_{k,1}})^{2r-i}\cdot(\hat{A}_{n_{k,1}}+\hat{B}_{n_{k,1}})^{2(j-r)+i+1} - (A_{n_{k,1}})^{2r-i}\cdot(\hat{A}_{n_{k,1}})^{2(j-r)+i+1}\Big\},$$

where the parameters $A_{n_{k,1}}$, $B_{n_{k,1}}$, $\hat{A}_{n_{k,1}}$ and $\hat{B}_{n_{k,1}}$, are defined in (11).

Further, we notice that for a Nakagami distribution

$$E\{V_k^{2j}\} = \frac{\Gamma(\epsilon+j)}{\Gamma(\epsilon)}\left(\frac{\nu_{0_k}}{m}\right)^j 2^j, \qquad (21)$$

where $\nu_{0_k} = E\{V_k^2/2\}$ is the average strength of the Nakagami faded path associate with the kth interfering user.

Having the moments of α_k the moments of α can be determined using the method described in [14].

Equations (19), (20) and (21) enable us to apply the MEM and MREM method to investigate the Multiple Access Interference.

B. Error Probability

Using (7) and standard techniques [15], it is easily shown that the conditional error probability for Coherent PSK can be derived as

$$P_{e|\alpha,\beta_1} = \frac{1}{2}\mathrm{erfc}\left\{\sqrt{\frac{E_b}{N_0}}(\alpha+\beta_1)\right\}. \qquad (22)$$

where

$$\mathrm{erfc}(\mu) = \frac{2}{\sqrt{\pi}}\int_\mu^\infty e^{-y^2}dy. \qquad (23)$$

To obtain an expression for the average error probability we can write (22) as

$$P_e = \int_{-\infty}^{\infty}\int_0^\infty P_{e|\alpha,\gamma_b}p(\gamma_b)p(\alpha)d\gamma_b d\alpha \qquad (24)$$

where

$$\gamma_b = \frac{E_b}{N_0}\sum_{i=1}^{P}\beta_i^2 \qquad (25)$$

with average

$$\overline{\gamma}_b = E\{\beta_i^2\}\frac{E_b}{N_0}, \qquad (26)$$

and $E\{\cdot\}$ denotes expected value.

Since β_1 is Nakagami distributed, it can be shown that γ_b is distributed according to

$$p(\gamma_b) = \left(\frac{m}{\gamma_0}\right)^\epsilon \frac{\gamma_b^{\epsilon-1}}{\Gamma(\epsilon)}\exp\left(-\frac{m\gamma_b}{\gamma_0}\right)\ \forall\gamma_b \geq 0. \quad (27)$$

The distribution $p(\alpha)$ in (22) is the MAI and is calculated using the moments derived in (19), (20), (21) and MEM. In the next section the MEM principles will be explained and it will become clear how $p(\alpha)$ is derived from the moments.

III. MAXIMUM ENTROPY BASED METHODS

Shore et al [16] have proven that MEM is the only method for inferring from incomplete information that does not lead to logical inconsistencies. Since the MAI can only be inferred from incomplete information, Maximum Entropy and Minimum Relative Entropy can be applied with success to calculate the MAI characteristics. MREM is closely related to MEM and therefore MEM is easily extended to determine the MREM. In the next two sections the necessary theoretical background is developed for the MEM and the MREM.

A. Maximum Entropy

In the MEM the missing information (the information entropy) [17]

$$I = -\int_a^b p(\alpha)\ln p(\alpha)d\alpha \qquad (28)$$

is maximized subject to the constraints of the normalization of the pdf and subject to the available information. In our case the expectation values of the moment operators must be equal to the measured or calculated moments. This is a standard maximum entropy moments problem which has been studied in great detail [18], [19]. The constraints are introduced via Lagrange multipliers and the resulting expression for the inferred pdf is

$$p(\alpha) = \frac{1}{Z}\exp\left\{-\sum_{m=1}^{M}\lambda_m x^m\right\} \qquad (29)$$

where the information about the normalization is contained in the partition function

$$Z = \int_a^b \exp\left\{-\sum_{m=1}^{M}\lambda_m\alpha^m\right\}d\alpha \qquad (30)$$

Once the Lagrange multipliers are solved using the moments of (19), (20) and (21) we obtain numerical values for the $\{\lambda_m\}$. All the information contained in the available moments (no more or no less) are used to determine the MAI distribution. From $\{\lambda_m\}$ a closed form expression of the MAI distribution is available in the form of (29)and can be used to calculate the average error rate. It is therefore not necessary to make any approximations to the nature of the MAI distribution anymore. Accurate error rates, as expressed by (24), can now be calculated using numerical integration. To solve the Lagrange multipliers we require that

$$\langle\alpha^m\rangle \equiv \int \alpha^m p(\alpha)d\alpha = N_m \quad ; \quad m = 1,\ldots,j \quad (31)$$

where N_m are the known moments of the MAI random variable (equations (19) (20) and (21)).

Alhassid et al. [20] have noted that defining

$$F(\{\lambda_m\}) = \ln Z + \sum_{m=1}^{M}\lambda_m N_m \qquad (32)$$

yields

$$\frac{\partial F}{\partial \lambda_m} = N_m - \langle \alpha^m \rangle. \tag{33}$$

Hence minimizing F is equivalent to solving the set of coupled nonlinear equations (31). Furthermore, the authors have shown that the Hessian matrix $\mathbf{H}$ with

$$H_{mm'} = \frac{\partial^2 F}{\partial \lambda_m \partial \lambda_{m'}} = \langle \alpha^{m+m'} \rangle - \langle \alpha^m \rangle \langle \alpha^{m'} \rangle \tag{34}$$

is positive definite and thus that F is a strictly convex function of the Lagrange multipliers $\{\lambda_m\}$. Consequently F has a unique minimum (i.e. (31) has a unique solution) and a Newton-Raphson minimization procedure [19] is guaranteed to converge. Define an error vector

$$\bar{\epsilon} = (\epsilon_1, \ldots, \epsilon_M)^T \quad : \quad \epsilon_m \equiv N_m - \langle \alpha^m \rangle \tag{35}$$

and let $\bar{\lambda} = (\lambda_1, \ldots, \lambda_M)$. Then the new guess after a Newton Raphson step is

$$\bar{\lambda}' = \bar{\lambda} - \mathbf{H}^{-1} \cdot \bar{\epsilon} \tag{36}$$

During each iteration we solve a set of coupled linear equations for the Newton step $\bar{\delta} = \bar{\lambda} - \bar{\lambda}'$

$$\mathbf{H} \cdot \bar{\delta} = \bar{\epsilon}. \tag{37}$$

Since $\mathbf{H}$ is positive definite it is also nonsingular. We solve equation (37) with a standard LU-decomposition with a back-substitution algorithm.

B. Minimum Relative Entropy

The discrimination information (relative entropy) between the MAI-distribution, $p(\alpha)$, (as inferred via MEM) and a Gaussian distribution, $q(\alpha)$, (with the same second order moment) can be derived as [7]

$$\mathcal{I} = \int_a^b p(\alpha) \ln \frac{p(\alpha)}{q(\alpha)} d\alpha, \tag{38}$$

The discrimination information is a measure of the evidence contained in the data, or moments, of $p(\alpha)$ discriminating against the Gaussian distribution q(a). I is therefore an information theoretic measure of the difference between two distributions.

IV. RESULTS

It is clear from (13) that the MAI random variable are dependent on the current data bit, b_0^k, and the previous data bit, b_{-1}^k, and therefore the correlation properties of the spreading sequences are also dependent from bit to bit. To substantiate this statement, Figure 1 shows a simulated EFR distribution of a CDMA system. The simulations were obtained with Gold spreading sequences under AWGN conditions, for $K = (10, 25)$ and $N = 63$. Fading were not considered in this case to isolate the MAI effect. When fading is present the errors will most likely be clustered and therefore obscure the effect of the MAI. Also indicated on the graph for comparative purposes, is the error free run of a Binary Symmetric Channel (BSC) at equivalent E_b/N_0. The reason for this comparison is that the errors on a BSC occur independently and any deviation from that can be easily spotted. From the graph it is clear that the CDMA system produces clustered errors, unlike the binary symmetric channel where errors occur independently. The flattening in the CDMA graphs indicate that there are long runs of bits without any errors and that, when errors occur, the errors occur in a bursty fashion. Since clustered (or burst) errors occur on channels with memory it is a plausible argument that the CDMA channel has indeed memory, especially when K is small. This fact also plays a significant role in selection of error control schemes for a CDMA system.

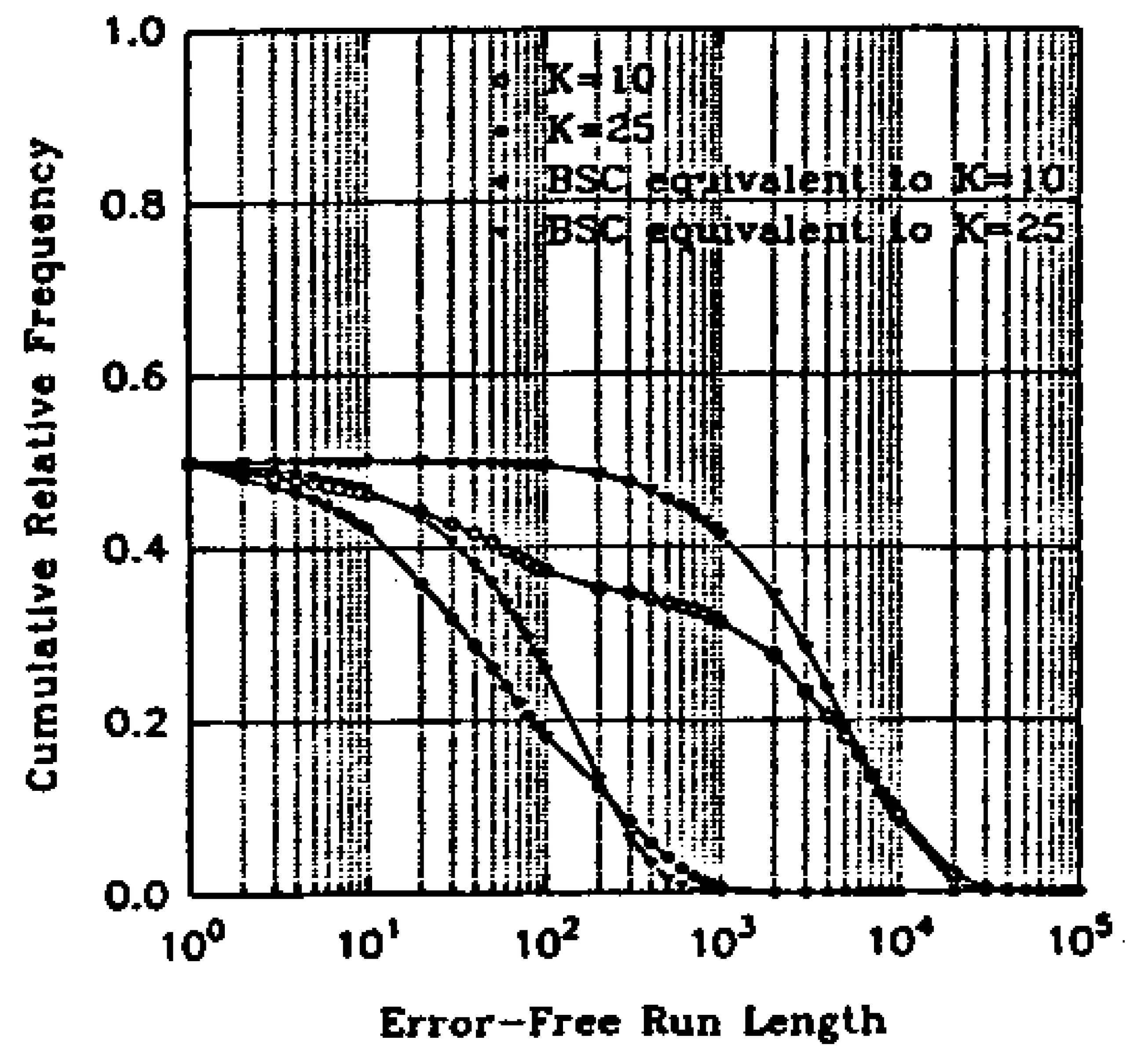

Fig. 1. Error free run of a simulated CDMA system with $K = \{10, 25\}$ and $N = 63$

The MAI pdf for $K = \{2, 25\}, N = 127$ and $m = \infty$ is as indicated in Figures 2 and 3 respectively. These distributions were generated using 13 moments of the MAI random variable. Only one side of the symmetric distribution is shown on a logarithmic scale.

For $K = 2$ (Figure 2) it is apparent that the distribution of the MAI is not very close to a Gaussian distribution. The MAI distribution has a lot more structure and does not follow the Gaussian tail at all. However, as intuitively expected the MAI random variable approaches the Gaussian distribution much better as K increases - as indicated in Figure 3 for $K = 25$ only the tail distribution of the MAI random variable does not follow the Gaussian distribution exactly. However, these tail probabilities

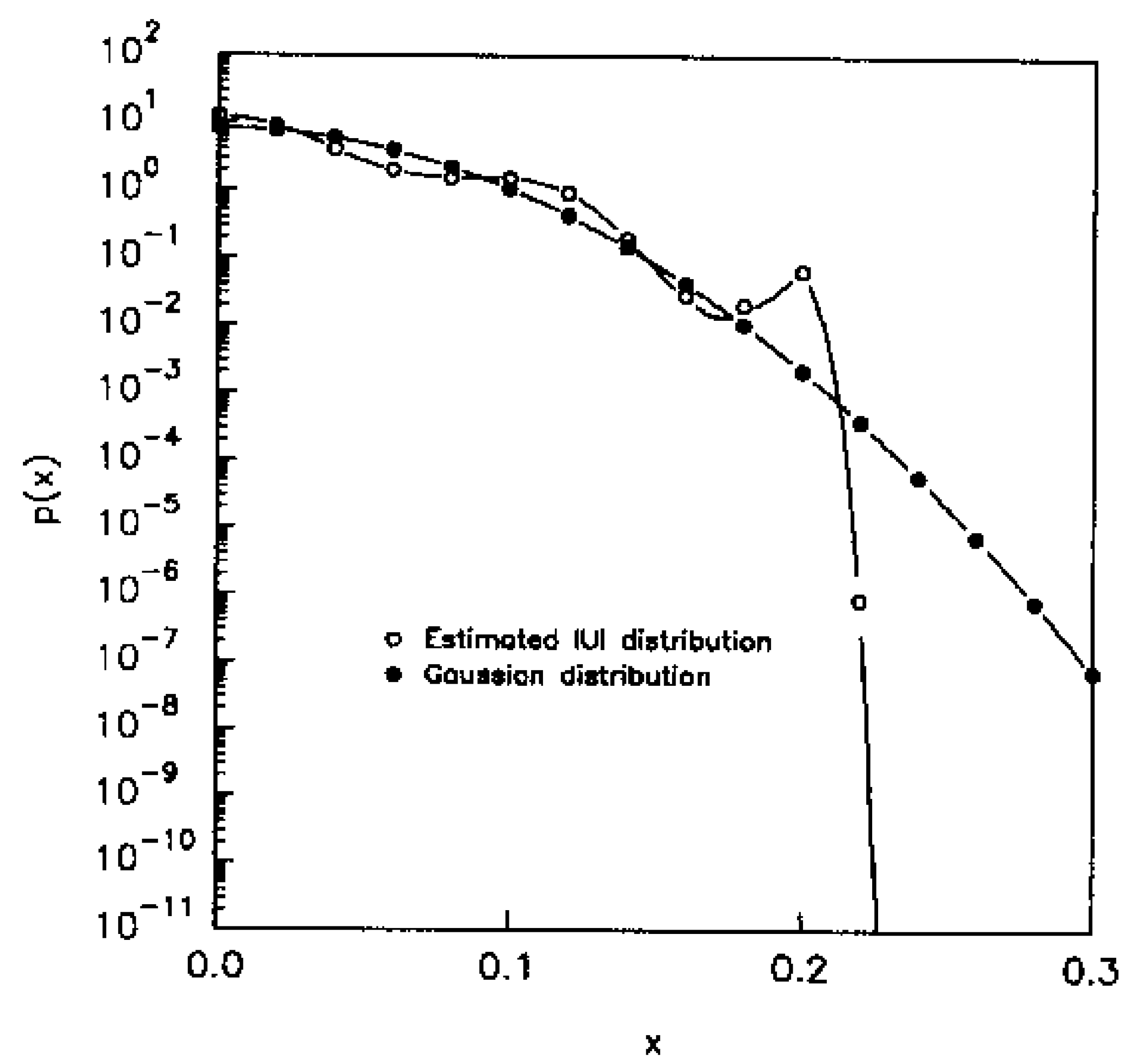

Fig. 2. MAI distribution for $K = 2, N = 127$, $N_m = 10$ and $m = \infty$

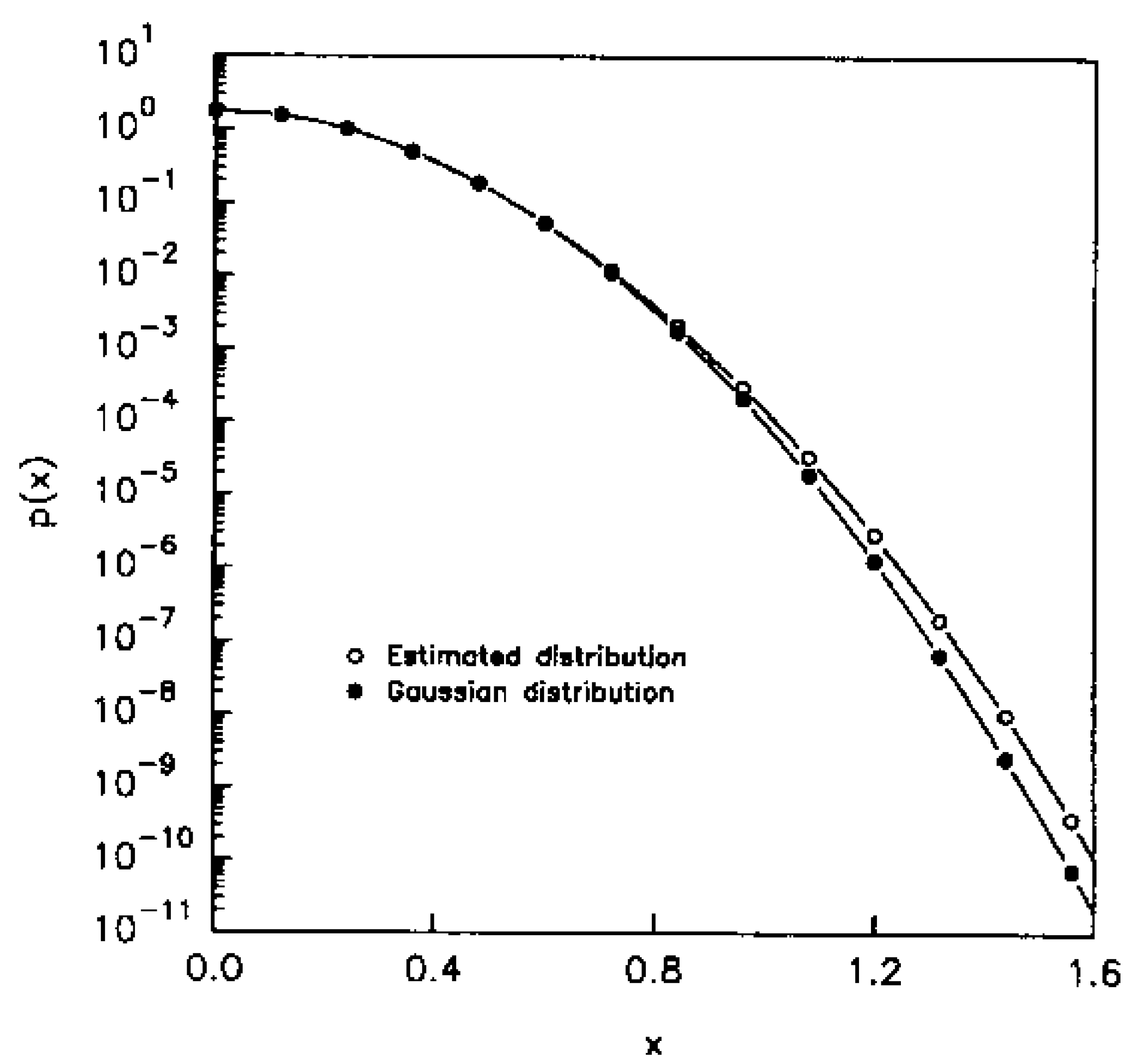

Fig. 3. MAI distribution for $K = 25, N = 127$, $N_m = 10$ and $m = \infty$

adversely affect the accuracy when average error probabilities are calculated.

By looking at the MAI distribution as in Figures 2 and 3, it is apparent that as K increases the GA becomes more valid. However, these measures, the EFR and the actual MAI pdf, do not give a generalized or quantitative indication of how close the MAI is to a Gaussian distribution. By calculating I, as defined in (38), we have a dimensionless quantity that fulfils the criteria of a generalized parameter to quantify the GA in an information theoretic manner. When $I = 0$ the two random variables are identically distributed.

Figure 4 shows I as calculated for the MEM inferred MAI pdf and a Gaussian distribution with the same second order moment for $m = \{\frac{1}{2}, 5, \infty\}$. Irrespective of m, the missing information between the two distributions decreases exponentially as K increases. As m decreases, in other words as the channel becomes more faded, the MAI distribution is less Gaussian. Further, for the Gold sequences adopted in this work, the maximum practical usable family size (balanced codes) is 65 for $N = 127$. From Figure 4 it is clear that the missing information, I, saturates at roughly 10^{-5} for $m = \infty$ and 10^{-3} for $m = \frac{1}{2}$. This observation is quite significant since an increase in the number of users, that is an increase in K, does not imply a more Gaussian nature of the MAI random variable under practical situations. The saturation of I suggest that the GA is only valid at high signal-to-noise ratios or high average error rates, especially under faded conditions. Another important conclusion from this is that in a cellular system, the intracell interference is likely to be under estimated when the GA is applied, especially under fading conditions. In these circumstances the MEM method can be used very effectively to determine the correct MAI distribution, and then to accurately predict the cellular capacity.

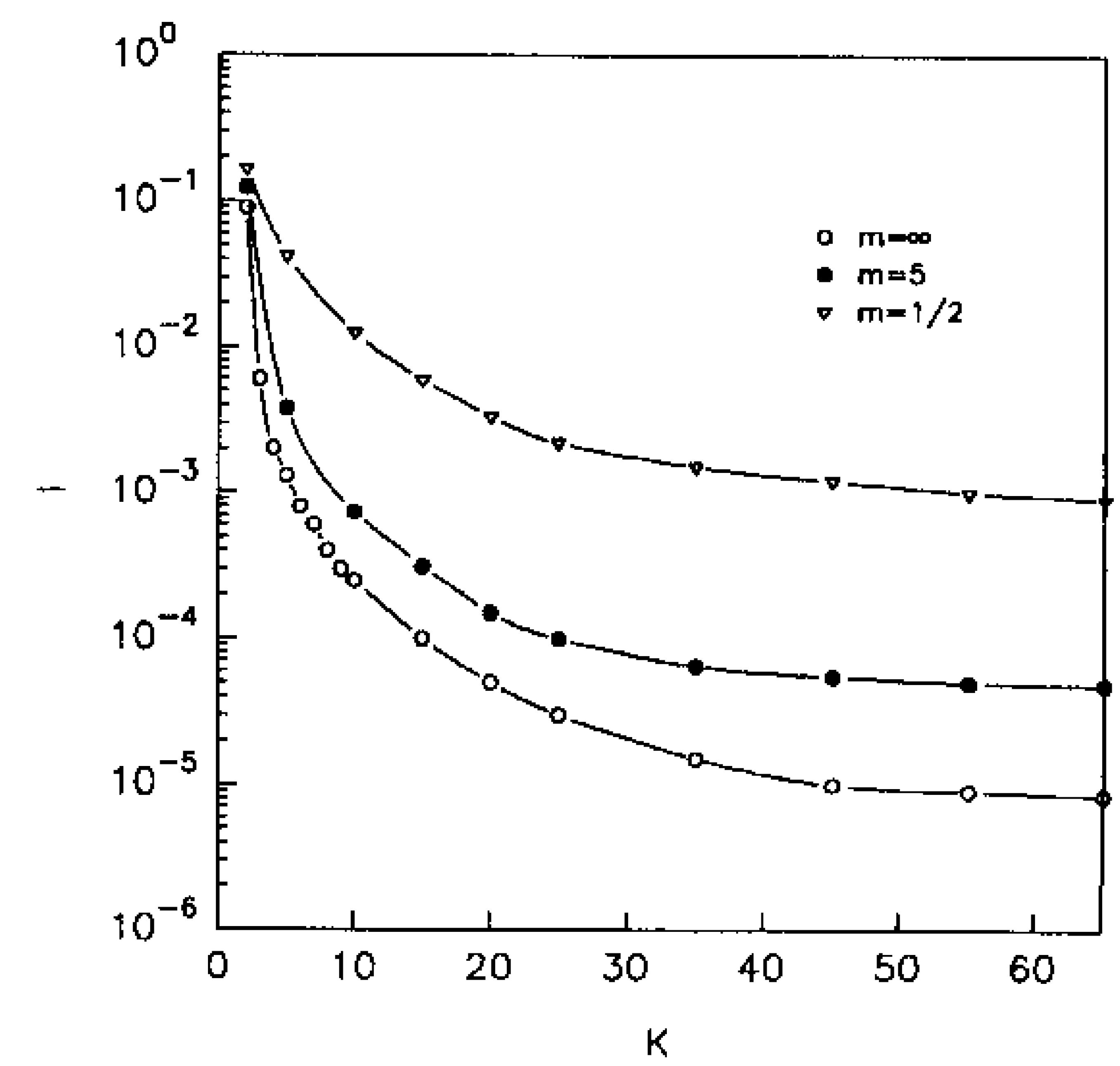

Fig. 4. Relative Entropy, $\mathcal{I}$, of the IUI distribution, $m = \{\frac{1}{2}, 5, \infty\}$

Table I gives an indication of the influence of the chipping sequence length N for $m = \{\frac{1}{2}, 5, \infty\}$. To investigate the influence of N the ratio $\frac{K}{N}$ has to be constant for different values of N. As N increases, I decreases, indicating that the MAI random variable becomes more Gaussian distributed as N increases. As for a continued increase in K, I also saturates at a specific value, indicating that not even an infinite increase in N will not result in a perfect Gaussian distribution. When fading is added to the signal, that is low values of m, the distribution is even less Gaussian.

It can be concluded that for the Gold spreading sequences used in this study, the GA is inaccurate, especially when a fading channel is considered. Having an accurate description of the MAI probability distribution function, (24) can be numerically integrated to obtain the average probability of error. Table II shows the accuracy of us-

		I		
K	N	$m = 1/2$	$m = 5$	$m = \infty$
2	31	0.12616	0.11971	0.01190
3	63	0.03171	0.02157	0.01154
5	127	0.01929	0.00613	0.00141
9	255	0.01348	0.00281	0.00049
17	511	0.00974	0.00128	0.00021

TABLE I

RELATIVE ENTROPY, I, FOR $m = \{\frac{1}{2}, 5, \infty\}$ AND $\frac{K}{N}$ CONSTANT

ing the MEM technique to calculate average error probability when no fading is present, i.e. $m = \infty$. The MAI distribution $p(\alpha)$ used to obtain these results areas is indicated in Figure 2. The bounds presented were obtained from [21]. MEM delivers excellent results, even for small values of K, indicating that average error rates can be calculated with great accuracy and that the MAI distribution derived by MEM is also accurate.

$\frac{E_b}{N_0}$	lower bound	upper bound	MEM
4	1.44×10^{-2}	1.45×10^{-2}	1.442×10^{-2}
6	3.35×10^{-3}	3.36×10^{-3}	3.345×10^{-3}
8	4.42×10^{-4}	4.24×10^{-4}	4.216×10^{-4}
10	2.40×10^{-5}	2.42×10^{-5}	2.399×10^{-5}
12	4.81×10^{-7}	4.89×10^{-7}	4.816×10^{-7}
14	2.28×10^{-9}	2.34×10^{-9}	2.285×10^{-9}

TABLE II

AVERAGE ERROR RATES FOR $K = 2$ AND $N = 31$

Using the MAI pdf derived with the MEM, Figure 5 shows some results for $K = 5$ and $N = 511$ for $m = (\frac{1}{2}, 1, 2, 3, 5)$.

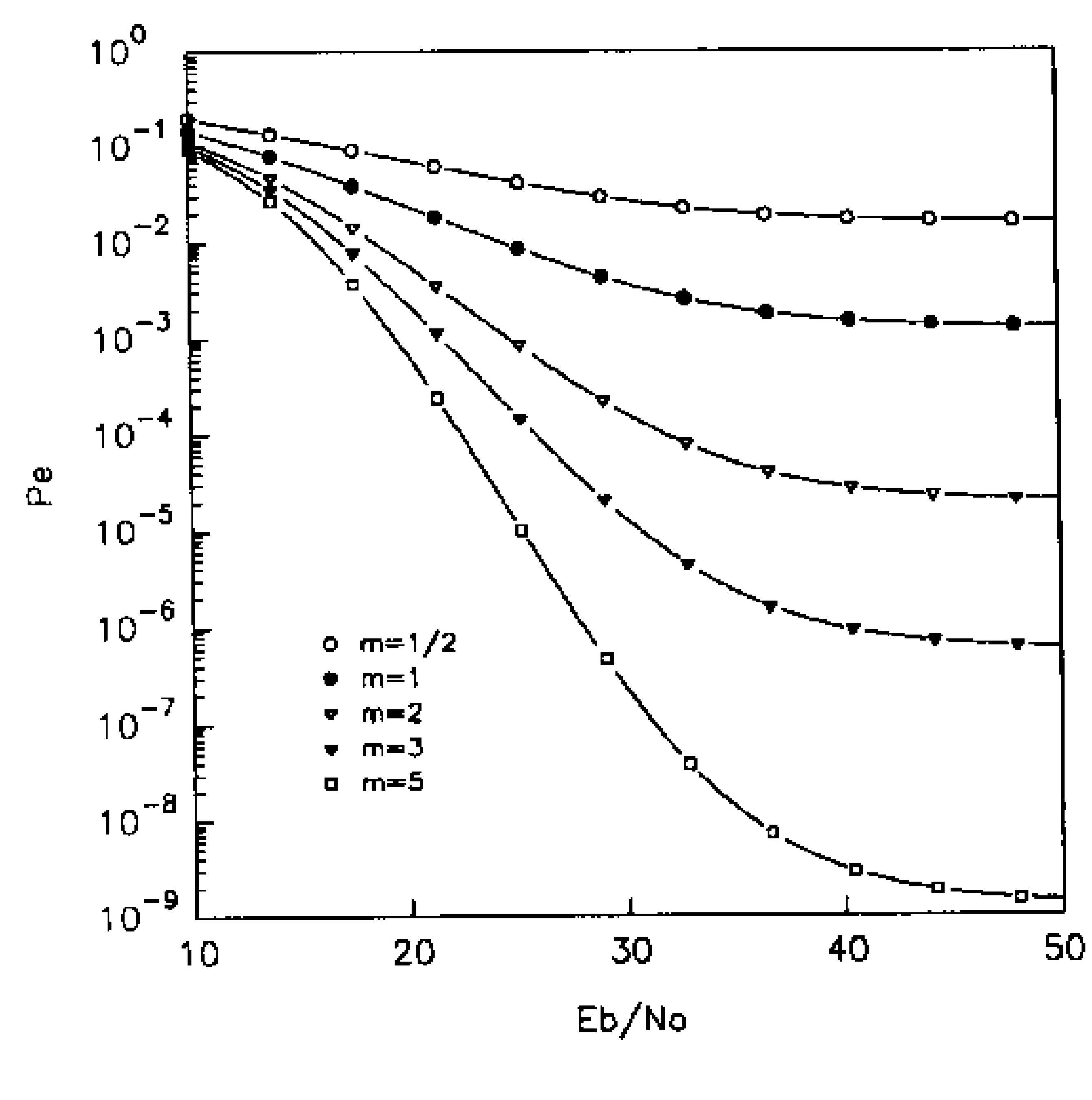

Fig. 5. Nakagami faded CPSK performance: $K = 5$, $L = 1$ and $N = 511$

V. CONCLUSIONS

The missing information between the MAI distribution and a Gaussian distribution with the same second moment were calculated and presented in a number of tables and graphs. The main conclusion to be drawn from the MAI investigation is that the missing information I for $m = 1/2$ (most severe fading) is almost three orders of magnitude more than for the unfaded case, $m = \infty$. Therefore, the Gaussian Assumption is significantly more valid under unfaded conditions, but more exact methods than the Gaussian Assumption have to be used to calculate error probabilities in faded CDMA systems.

The most significant aspect of this paper is that the MEM and MREM techniques were presented to investigate the MAI in a CDMA system. It has further been shown that apart from an accurate description of the MAI very accurate average error rates can also be obtained by this technique. Closed form expressions for the MAI distribution can be obtained using (29), making it possible to derive accurate results for CDMA for virtually any modulation scheme, making it unnecessary to rely on inaccurate assumptions, such as the Gaussian Assumption. In fact, the MEM method can be applied to any problem where the exact distribution of the random variable is not available, but power moment information can be obtained. An example of such a system are intersymbol interference problems where the intersymbol interference random variable moments can be calculated. Also, using MREM principles, the missing information can be used as an error signal to update adaptive algorithms when, for instance, interference cancellation is required. It would also be interesting to see how MREM compare with other adaptive updating techniques such as the LMS algorithm. One of the future aspects that will be considered is the influence of non-ideal power control on the MAI distribution.

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