Low Rate coding and Diversity for Cellular CDMA

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Abstract: This paper considers the performance of direct sequence cellular Code Division Multiple Access (CDMA) with low rate convolutional coding and diversity. The communications channel is assumed to be multipath faded, and modeled as a discrete set of Nakagami faded paths. The Nakagami fading distribution has been shown to fit empirical results more generally than other distribution [1]. New analytical results to evaluate the probability of error for the receiver terminals studied in this work are presented. The analytical expressions to calculate the average error rate make us of the Gaussian assumption. Numerical results reveal that, for non-diversity receivers, Nakagami fading is very hostile to a CDMA system, reducing the capacity substantially. However, with the use of RAKE reception, interleaving and convolutional coding, the average error rate can be reduced dramatically, allowing for reliable communications.

I. INTRODUCTION

With the introduction of the Universal Mobile Telecommunication Service (UMTS) early in the next century, powerful coding and performance enhancing techniques are needed to fulfill the required system capacity. Spread spectrum signaling techniques, with its inherent anti-multipath, multiple access and rejection of interference capabilities, in the form of Wideband CDMA (WCDMA) and Time Division CDMA (TD-CDMA), was chosen as the multiple access technology for UMTS in January 1998. Until recently the standard analysis of CDMA systems was rather pessimistic about the capacity of these systems compared to FDMA and TDMA. Gilhousen et al [2] recognized that since CDMA capacity is only interference limited (unlike FDMA and TDMA) any reduction in interference converts directly and linearly into an increase in capacity. Therefore, by employing voice activity factor, sectorizing the cells and using various forms of diversity it is possible to achieve CDMA system capacity at least as good as FDMA and TDMA. This improvement has been indicate by [2] and others under AWGN conditions. In this paper a detailed analysis of the performance of a cellular CDMA system with diversity under frequency-selective slowly fading Nakagami and multipath conditions is presented. For diversity purposes RAKE reception and convolutional coding are considered. Coherent PSK is assumed a modulation scheme and consequently the analysis can be applied to the downlink of a cellular SSM system. There is a sizable literature relating to the effects of multiple access interference on the performance of cellular CDMA, among which are [3] and [4]. Most literature, except Xiang [3], assume either Rayleigh or Rician fading statistics to model the communication channel. It has been shown, however, that the Nakagami-m distribution assumes the signals are received with random moduli and phases, leading to more flexibility in matching experimental data than that of Rayleigh or Rice models [5], [6]. This is especially true for the indoor wireless and densely built urban channels Using the equations derived in this work it is possible to predict CDMA capacity under the mentioned conditions with RAKE reception and convolutional coding. By introducing a voice activity factor of 3/8 and cell splitting of 3, the performance of a cellular system is assessed. It is assume that hard decisions are made by the demodulator and that the error- producing mechanism result in independent error events. The latter assumption requires interleaving at the transmitter an dc-interleaving at the receiver. Section II analyses and describes a system model for a typical indoor wireless communication channel The analyses allows for the calculation of the average error probabilities by means of closed for expressions, making use of the Gaussian Assumption to model the multiple access interference [7]. The bounds used to calculate the convolutional coded performance is presented in Section III. Numerical results are discussed in Section IV and finally, conclusions are presented in the last section of the paper.

II. SYSTEM MODEL AND EVALUATION

The model considered will be summarized briefly and is based on the model developed by Kavehrad [4]

Measurements by Qualcomm [8] indicate that the adjacent cell interference in a cellular system contribute approximately 50% of the total interference (for equally loaded cells). Therefore, assuming equally loaded cells, the maximum number of users a cellular system can support is given by

$$K = K' \frac{N_{sect}}{V_{on}} 1.5 \tag{1}$$

where K is the total number of users per cell, V_{on} the voice activity factor and N_{sect} the cell splitting factor. Our conjectural system will assume $\frac{N_{sect}}{V_{on}} = 8$. An equivalent spread spectrum multipath system model for K users is indicated in Figure 1. The channel for the desired transmitter and receiver ($k = 1$) can be represented by an L-paths Nakagami fading model where a single transmitted pulse is received via L-paths at the random instant $t_l, l = 1, \cdots, L$. We assume t_l is uniformly distributed over one bit period $(0, T)$ and that each user coding sequence has a period of $N = T/T_c$.

In the analysis we assume that average power control is assumed which also includes averaging the channel fading characteristics. Baseband signaling at a rate less than the channel coherence bandwidth ensures that intersymbol interference can be neglected. Therefore, the channel has low-pass equivalent impulse response, given by

$$h(t) = \sum_{l=1}^{L} \beta_l \delta(t - t_l) e^{j\phi_l}, \tag{2}$$

where $\delta(\cdot)$ is the delta function, β_l is the Nakagami distributed path gain and ϕ_l is the random path phase, uniformly distributed between $(0, 2\pi]$. In the transmission model it is further assumed that the kth interfering user of the multiple access system is linked to the receiver of Figure 1 via a single Nakagami fading path with a uniformly distributed random delay τ_k ranging from zero to one bit period, T. This will naturally result in a worst case scenario, rendering the results conservative. In our formulation we specify the Nakagami distributed path gain of the $K-1$ interfering users by $V_q, k = 2, \cdots, K$. Thus, the received signal for the fading model described is given by

$$\begin{aligned} r(t) &= A \sum_{l=1}^{L} \beta_l a_1(t - t_l) b_1(t - t_l) \cos(\omega_c t + \Phi_l) \quad (3) \\ &+ A \sum_{k=2}^{K} V_k a_k(t - \tau_k) b_k(t - \tau_k) \cos(\omega_c t + \Psi_k) \\ &+ n(t), \end{aligned}$$

where $\Phi_l = \theta_1 - \omega_c t_l + \phi_l$, $\Psi_k = \theta_k - \omega_c \tau_k$ and θ_k the phase of the kth user. Also, $n(t)$ is white Gaussian noise with double sided spectral density of level $N_0/2$ and θ_1 can be assumed zero with no loss of generality. Since coherent PSK is considered, the receiver is assumed to coherently recover the carrier phase and delay lock to the first arriving desired signal. Following Kavehrad [4] and standard procedures, after matched filter reception, the conditional probability of error is given by

$$P_{e|\alpha,\beta_1} = \frac{1}{2} \text{erfc} \left\{ \sqrt{\frac{E_b}{N_0}} (\alpha + \beta_1) \right\}. \tag{4}$$

where

$$\begin{aligned} \alpha &= x + y, \quad (5) \\ x &= \sum_{k=2}^{K} \frac{V_k}{T} \left\{ b_{-1}^k R_{k,1}(\tau_k) + b_0^k \hat{R}_{k,1}(\tau_k) \right\} \cos \Theta_k, \\ y &= \sum_{l=2}^{L} \frac{\beta_l}{T} \left\{ b_{-1}^1 R_{1,1}(t_l) + b_0^1 \hat{R}_{1,1}(t_l) \right\} \cos \Psi_l. \end{aligned}$$

and b_0^1 represents the information bit being detected and b_{-1}^1, is the preceding bit, which, due to the channel delay spread, affects the detection of b_0^1 received on the first path between the desired transmitter and receiver. The parameters β_l and V_k are sample values of a Nakagami variable. The discrete partial auto- and cross correlation functions are given by

$$\begin{aligned} R_{k,1}(\tau) &= A_{n_{k,1}} T_c + B_{n_{k,1}} (\tau - nT_c) \quad (6) \\ \hat{R}_{k,1}(\tau) &= \hat{A}_{n_{k,1}} T_c + \hat{B}_{n_{k,1}} (\tau - nT_c) \quad (7) \end{aligned}$$

and, together with the variables $A_{n_{k,1}}$, $B_{n_{k,1}}$, $\hat{A}_{n_{k,1}}$ and $\hat{B}_{n_{k,1}}$, defined in [9], enable us to evaluate the system performance for specific code parameters. In (4) β_1 is Nakagami distributed and α is the IUI variable, which is assumed to be Gaussian distributed. The conditioning in α is removed analytically by realising

$$P_{e|\beta_1} = \int_{-\infty}^{\infty} P_{e|\alpha,\beta_1} p(\alpha) d\alpha, \tag{8}$$

where

$$p(\alpha) = \frac{1}{\sqrt{2\pi}\sigma_{ma}} \exp\left(-\frac{\alpha^2}{2\sigma_{ma}^2} \right). \tag{9}$$

and σ_{ma}^2, is the multiple access interference variance. Integrating by parts, using integration tables by Ng et al [10] and some manipulations, (8) results in

$$P_{e|\beta_1} = \frac{1}{2} \text{erfc} \left\{ \sqrt{\Lambda \beta_1^2 \frac{E_b}{N_0}} \right\}, \tag{10}$$

with

$$\frac{1}{\Lambda} = 1 + 2 \frac{E_b}{N_0} \sigma_{ma}^2. \tag{11}$$

Next the conditioning in β_1 has to be removed. Using the channel model described earlier, where the channel is modeled as a P tap delay line, we can derive the performance of an optimum RAKE receiver. By definition β_1 as being Nakagami distributed with actual received signal-to-noise ratio

$$\gamma_b = \frac{E_b}{N_0}\sum_{i=1}^{P}\beta_i^2 = \sum_{i=1}^{P}\gamma_i, \qquad (12)$$

with average

$$\overline{\gamma}_b = E\{\beta_i^2\}\frac{E_b}{N_0}, \qquad (13)$$

where $E\{\cdot\}$ denotes expected value. Each of the $\{\gamma_i\}$ is distributed according to

$$p(\gamma) = \left(\frac{m}{\gamma_0}\right)^m \frac{\gamma^{m-1}}{\Gamma(m)}\exp\left(-\frac{m\gamma}{\gamma_0}\right) \qquad (14)$$

Using the characteristic function of (14) and the fact that the P channels are mutually statistically independent, the probability distribution function of $p(\gamma)$ is

$$p(\gamma_b) = \left(\frac{m}{\gamma_0}\right)^\epsilon \frac{\gamma_b^{\epsilon-1}}{\Gamma(\epsilon)}\exp\left(-\frac{m\gamma_b}{\gamma_0}\right) \quad \forall \quad \gamma_b \geq 0. \qquad (15)$$

$\epsilon = mP$, m the Nakagami fading parameter, and P the number of taps in the RAKE receiver. Since all P channels are assumed to be equal and independent, the mean-square values $E\{\beta_i^2\}$ will be equal, and therefore, (12) can be written as

$$\gamma_b = P\beta_1^2\frac{E_b}{N_0}. \qquad (16)$$

Substituting (16) into (10), the average error rate can be determined by

$$P_e = \int_0^\infty P_{e|\gamma_b}p(\gamma_b)d\gamma_b. \qquad (17)$$

The result after integration by parts, using tables in [10] and Abramowitz et al [11], we derive at

$$P_e = \frac{1}{2}\left\{1 - \frac{2\Gamma\left(\epsilon+\frac{1}{2}\right)}{\sqrt{\pi}\Gamma(\epsilon)}\sqrt{\frac{\Lambda\overline{\gamma}_b}{m}}\ {}_2F_1\left(\frac{1}{2};\epsilon+\frac{1}{2};\frac{3}{2};-\frac{\Lambda\overline{\gamma}_b}{m}\right)\right\} \qquad (18)$$

where ${}_2F_1(\cdot)$ is the Gauss hypergeometric function [11]. We note that for $m = 1$ and $P = 1$, which is single path Rayleigh fading, ${}_2F_1(\cdot)$ reduces to [11]

$${}_2F_1\left(\frac{1}{2};\frac{3}{2};\frac{3}{2};-q\right) = (1+q)^{-\frac{1}{2}}. \qquad (19)$$

Substituting (19) in (18), realising that $A = 1$ in the absence of IUI and some manipulations, (18) reduces to

$$P_e = \frac{1}{2}\left\{1 - \sqrt{\frac{\gamma_0}{1+\gamma_0}}\right\} \qquad (20)$$

which is the ideal performance of a single-path Rayleigh fading channel [12]. The appendix gives a derivation of (18) for the unfaded case; that is when $m = \infty$.

III. CHANNEL CODING

Error control coding can be used with great success in CDMA with no penalty paid in bandwidth by the addition of redundancy to the information bits. It is further shown that low rate coding can be used very efficiently in a CDMA system. In this work rate $R_{cd} = \{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}\}$ constrained length $\nu = \{2, 3, 4\}$ convolutional codes are considered. It is assumed that the PN spreading sequence spans one code symbol. This implies that under the assumption of fixed throughput (i.e. constant data rate), fixed maximum chip rate and fixed complexity, a rate R_{cd} code must employ a PN spreading sequence shorter by a factor R_{cd} that of the uncoded case. This results in increased interuser interference due to the poorer crosscorrelation properties of shorter PN sequences. In our case we use PN sequences of $N = 511$ for the uncoded case and $N = 255, 127$ and 63 for rate $R_{cd} = \{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}\}$ codes respectively. The Chernoff bound, as indicated in [12], is used to derive performance results, and is given by

$$P_b \leq \sum_{d=d_{free}}^{\infty} a_d P_2(d), \qquad (21)$$

with $P_2(d)$ is the pairwise error probability as indicated in [12] and the coefficients $\{a_d\}$ related to the number of paths corresponding to the set of distances $\{d\}$.

IV. NUMERICAL RESULTS

In the subsequent evaluations, with no loss of generality, the normalization $\Omega = -10dB$ is adopted s that the received energy in the fading channel, $\beta_1^2 E_b$, has an average value $E\{\beta_1^2 E_b\} = \Omega E_b = 0.1E_b$. With $\Omega = 0.1$, the Nakagami pdf is reduced to a one-parameter distribution so that all the result can be expressed in terms of the single parameter m. To investigate the influence of the Nakagami parameter m, six values are investigated; $m = \{\frac{1}{2}, 1, 2, 3, 5, \infty\}$. For $m = 1$ the Nakagami pdf reduce to the Rayleigh pdf and hence can be used for benchmark purposes and for $m = \infty$ the Nakagami pdf tends to an impulse response and therefore represents the unfaded case. The parameter m is inversely proportional to the amount of fading. In other words, as m increases the pdf tends to a impulse response and consequently represents no fading.

A. Nakagami Performance without Diversity

Figure 2 indicates the performance for $K = 2$ and $L = l$. It is apparent that the average error rate decreases substantially as m increases. For $m = \frac{1}{2}$, severest fading, the performance for as little as two users is very poor and saturates at an error rate of 10^{-2} and consequently the system fails as a multiple access system for $m < 1$.

Table 1 indicates the capacity of a Nakagami faded CDMA system for different values of m and

L at $E_b/N_0 = 50dB$. The case $m = \infty$ represents the unfaded performance and also the value that the capacity of a Nakagami faded channel will saturate to. Using Table 1 and (1) it is possible to calculate the capacity of a cellular system with voice activity monitoring and cell splitting. It is clear that by virtue of a cellular architecture it is possible to increase the system capacity by a factor of five. However, the capacity of such a system is still relatively low and it is clear that some form of diversity is needed to substantially increase the system capacity, especially under multipath fading conditions.

	P_e	$L=1$	$L=5$	$L=10$
$m=1/2$	10^{-3}	-	-	-
$m=1$	10^{-3}	4	-	-
$m=2$	10^{-3}	30	25	18
	10^{-5}	3	-	-
$m=3$	10^{-3}	57	52	45
	10^{-5}	11	6	-
$m=5$	10^{-3}	90	84	76
	10^{-5}	27	22	16
	10^{-8}	7	-	-
$m=\infty$	10^{-3}	152	-	-
	10^{-5}	77	-	-
	10^{-8}	41	-	-

TABLE I

K FOR NON-DIVERSITY CPSK: $E_b/N_0 = 60$ DB, $P = 1$ AND $L = \{1, 5, 10\}$

B. Nakagami Performance with RAKE Reception

Figures 3 and 4 indicate the performance improvement by using a RAKE receiver as a function of the number of multiple paths, L, and average error rate for $K = 30$. For $m = \frac{1}{2}$ and a five tap RAKE receiver the average error rate is decreased by approximately four orders of magnitude. Interesting to note is the linear increase in average error rate as L increases. This is as expected since each multipath ray, delayed by more than one chip of the spreading code, can be viewed as an additional user, and it is well known that the multiple access interference noise has linear characteristics [2]. As benchmark, the performance for $m = 1$ is as indicated in Figure 4. Again the performance is substantially improved as the number of taps, P, increases. For a five tap RAKE receiver with $m = 1$, the performance is approximately three orders better than for the $m = \frac{1}{2}$ case. Table 2 indicates the capacity of a single cell Nakagami faded system with RAKE reception up to four taps. A linear increase in K with P is noticeable. As before, the multiple cell capacity can be obtained by a factor of five when voice activity and sectorization is included.

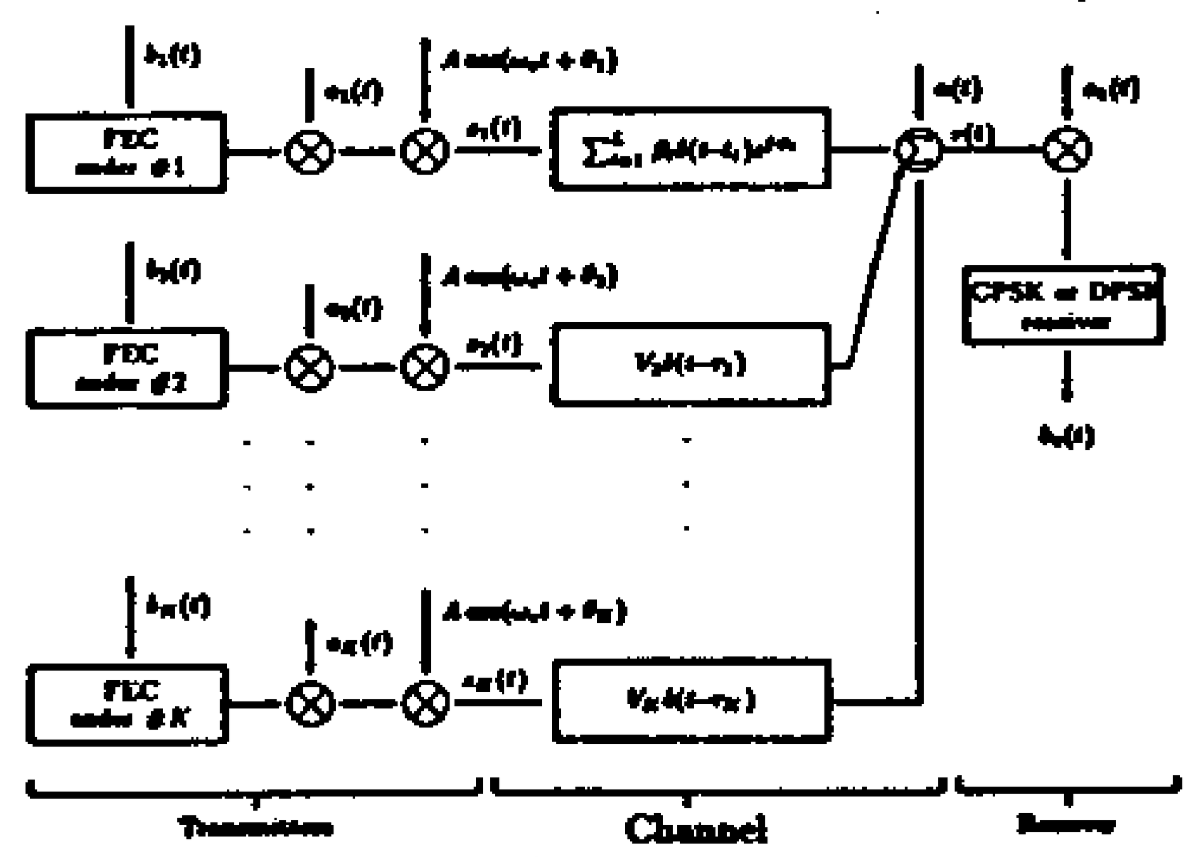

Fig. 1. System model

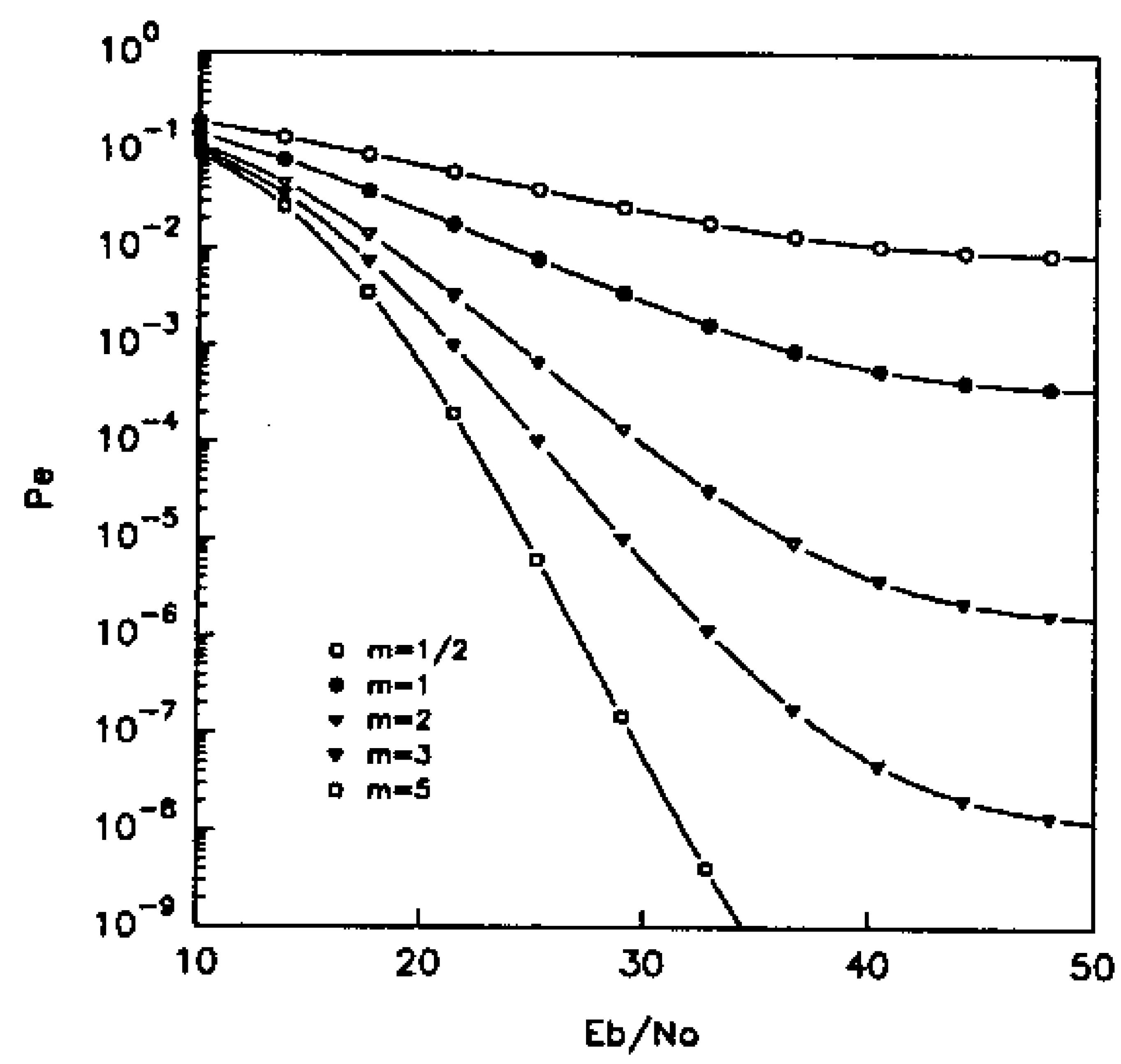

Fig. 2. Non-diversity CPSK performance, $K = 2, L = 1, P = 1$

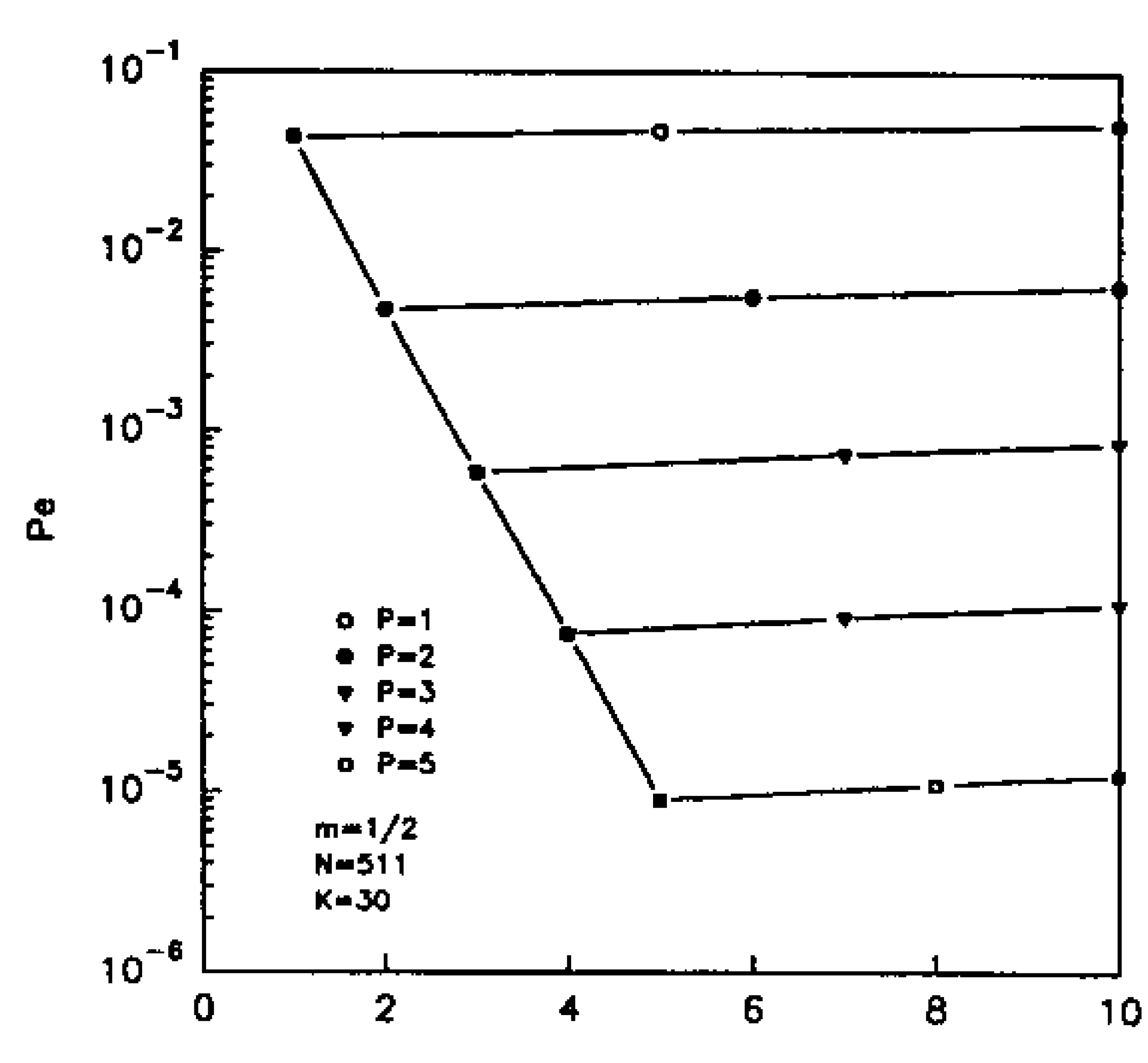

Fig. 3. Average error rate as a function of the number of channel paths: $m = \frac{1}{2}$, $K = 30$ and $N = 511$

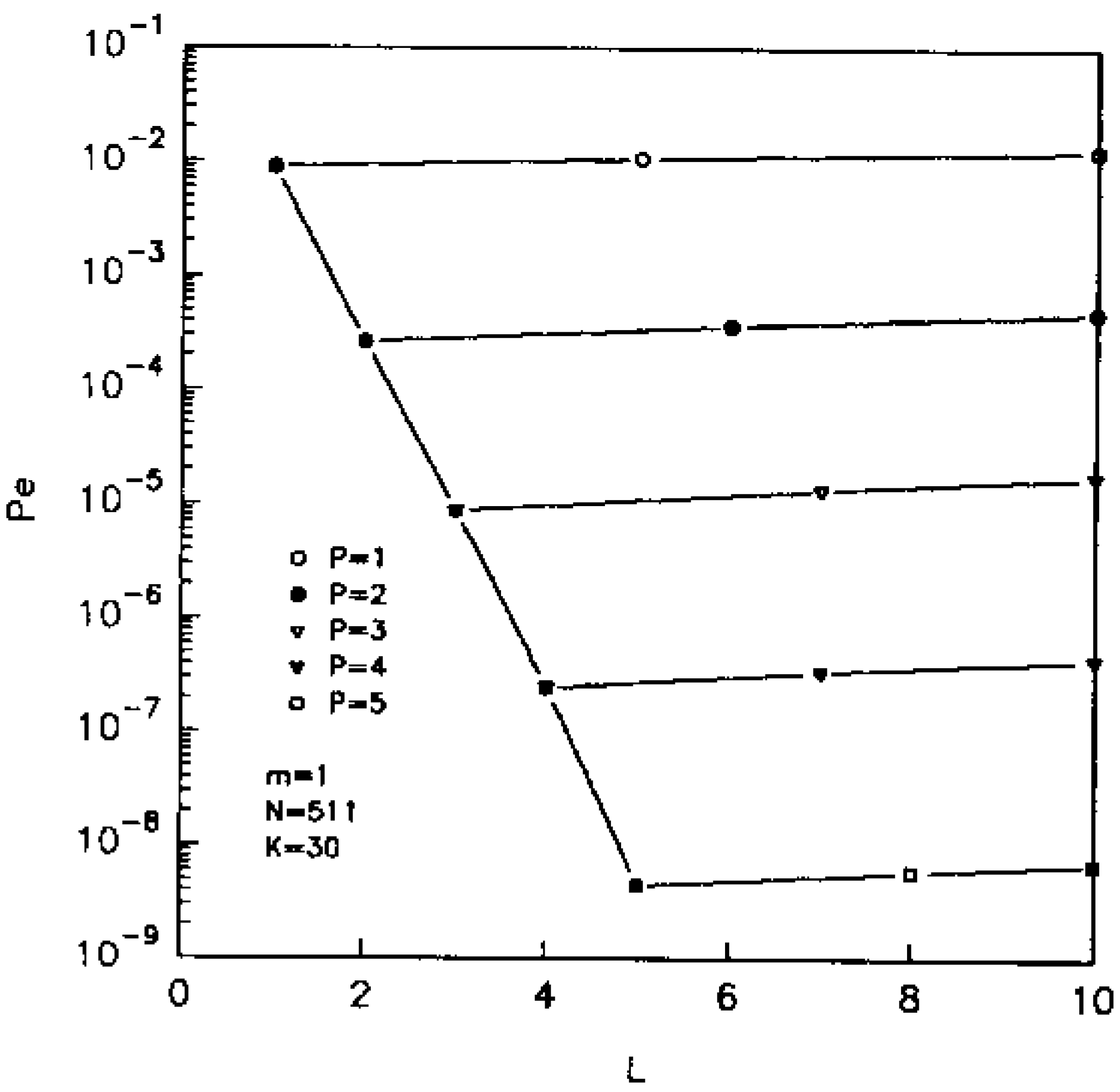

Fig. 4. Average error rate as a function of the number of channel paths: $m = 1$, $K = 30$ and $N = 511$

	P_e	$P=2$	$P=3$	$P=4$	$P=5$
$m = 1/2$	10^{-3}	5	7	115	215
	10^{-5}	-	-	8	27
$m = 1$	10^{-3}	59	167	298	440
	10^{-5}	5	30	74	130
	10^{-8}	-	-	9	25
$m = 2$	10^{-3}	150	295	447	601
	10^{-5}	38	98	169	244
	10^{-8}	6	25	55	91
$m = 3$	10^{-3}	191	350	505	662
	10^{-5}	66	138	215	294
	10^{-8}	17	49	87	129
$m = 5$	10^{-3}	242	398	556	714
	10^{-5}	99	177	258	340
	10^{-8}	37	78	122	167

TABLE II

K FOR RAKE RECEIVED CPSK: $E_b/N_0 = 60$ dB, $P = \{2,3,4,5\}$ AND $L = \{2,3,4,5\}$

	P_e	$R_{cd} = 1/2$	$R_{cd} = 1/4$	$R_{cd} = 1/8$
$m = 1/2$	10^{-3}	9	27	98
	10^{-5}	-	8	42
	10^{-8}	-	2	15
$m = 1$	10^{-3}	57	65	163
	10^{-5}	19	30	87
	10^{-8}	4	11	43
$m = 2$	10^{-3}	125	94	207
	10^{-5}	61	52	119
	10^{-8}	23	26	67
$m = 3$	10^{-3}	156	106	223
	10^{-5}	85	61	131
	10^{-8}	40	34	77
$m = 5$	10^{-3}	185	116	236
	10^{-5}	109	69	141
	10^{-8}	59	40	85
$m = \infty$	10^{-3}	226	256	270
	10^{-5}	141	156	160
	10^{-8}	86	95	98

TABLE III

K FOR CONVOLUTIONAL CODED CPSK WITH $E_b/N_0 = 60$ dB, $P = 1$, $L = 1$, $R_{cd} = \frac{1}{4}$ AND $N = 127$

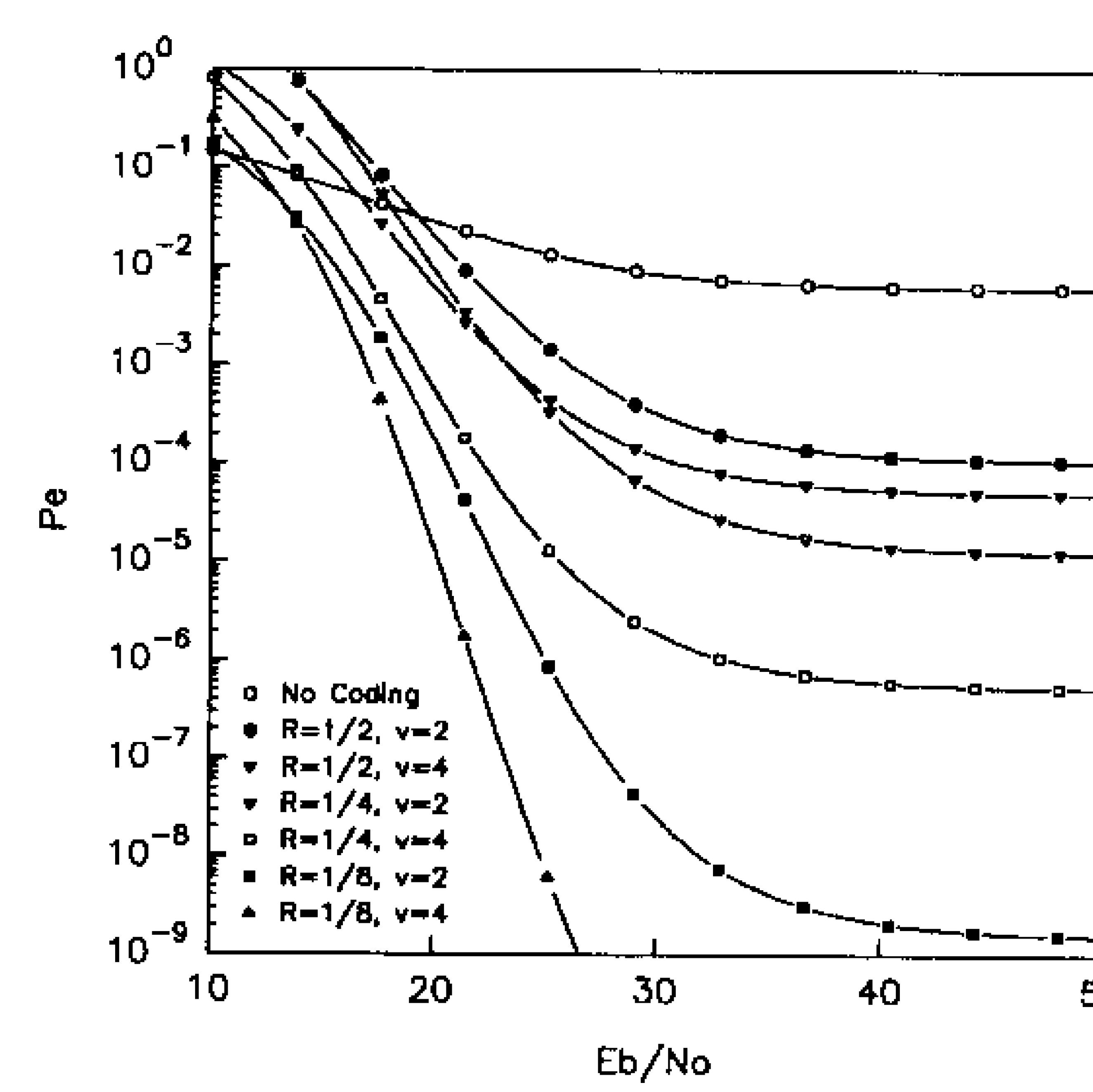

Fig. 5. Convolutional coded CPSK performance: $K = 20$ and $m = 1$

C. Nakagami Performance with Coding

As another form of diversity, we consider the performance of a CDMA system under Nakagami fading with convolutional coding. As mentioned before coding can be implemented very efficiently in a spread spectrum system. Table 3 indicates the capacity improvement due to rate $R_{cd} = \{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}\}$ convolutional codes with constrained length $\nu = 4$. Even with $m = \frac{1}{2}$ it is possible to have nine users at an error rate of 10^{-3}. Figure 5 indicates the performance for $m = 1$ and $K = 20$.

D. Nakagami Performance with Coding and RAKE reception

Tables 4 indicates the capacity for $R_{cd} = \frac{1}{2}$ and $P = (2,3,4,5)$. It is clear that a combination of convolutional coding and RAKE reception is very effective in increasing the system capacity. For example, by introducing a $R_{cd} = \frac{1}{2}$ convolutional code with a $P = 5$ tap RAKE receiver, the capacity improvement over a non-diversity receiver can be increased by almost three orders to 915 users for $m = 1$ at $P_e = 10^{-3}$ a massive increase. Even more important, for $m = \frac{1}{2}$ the system fails as a multiple access scheme when no diversity is used. Using both convolutional coding and RAKE reception the capacity can be increased to 708 at an error rate of 10^{-3} with a commercially available convolutional coder and a five tap RAKE receiver.

V. CONCLUSIONS

From the numerical work presented in this paper the following conclusions can be drawn. If all discrete paths have Nakagami gains with low values of m and guaranteed low average error probability is expected at all times, a simple non-diversity coherent receiver will not be able to cope with the Nakagami channel fading with binary spread spectrum codes of period $N = 511$. In fact, even for

	P_e	$P=2$	$P=3$	$P=4$	$P=5$
$m=1/2$	10^{-3}	112	289	493	708
	10^{-5}	35	123	238	367
	10^{-8}	5	35	87	154
$m=1$	10^{-3}	247	466	691	919
	10^{-5}	120	253	395	540
	10^{-8}	45	116	198	286
$m=2$	10^{-3}	346	575	805	1037
	10^{-5}	198	345	494	644
	10^{-8}	100	189	281	375
$m=3$	10^{-3}	384	615	846	1078
	10^{-5}	230	380	530	680
	10^{-8}	127	219	313	408
$m=5$	10^{-3}	416	647	879	1111
	10^{-5}	258	409	560	711
	10^{-8}	151	246	341	436

TABLE IV

K FOR CONVOLUTIONAL CODED AND RAKE RECEIVED CPSK AT $E_b/N_0 = 60$ dB, $P = \{2,3,4,5\}$, $L = \{2,3,4,5\}$, $R_{cd} = \frac{1}{2}$, $\nu = 4$ AND $N = 255$

the unfaded $m = \infty$ case the capacity is unacceptably low. By using longer sequences to decrease the error probability of the interference limited system, this problem can be partly overcome. However, increasing the spreading code's length is not always a viable option due to restricted bandwidth. The results indicate that in the absence of diversity even small amounts of multiple access interference can be harmful in a Nakagami fading environment when m is small. However, when either or both RAKE reception and error control coding are introduced the performance and capacity of our CDMA system can be increased dramatically. Also, by introducing voice activity monitoring and a cellular architecture, the capacity as tabulated, can be increased by a factor of five. Another important result is that low rate convolutional codes can be used to increase the system capacity dramatically, without a penalty paid in bandwidth. This is fundamentally different when compared to narrowband systems. From the performance results it is further evident that error control coding combined with RAKE reception are power and bandwidth efficient. All the capacity assessments were made at $\frac{E_b}{N_0} = 60dB$. However, using these forms of diversity the average error rate saturates at much lower values of signal- to-noise ratio and therefore the transmitted power needed for a given number of simultaneous users can be much lower. This is significant, especially in handhold units where power consumption is critical. To conclude, to guarantee acceptable performance under Nakagami fading conditions some form or forms of diversity is absolutely necessary - the type of diversity needed is to be dictated by the specific application and cost considerations.

VI. Appendix

As mentioned, $m = \infty$ corresponds to an unfaded signal. Consequently, since $L = 1$, a RAKE receiver would not constitute any advantage, and therefore P and ϵ are equal to one and m respectively for this condition. Now, taking the limit of $m \to \infty$ in (18) and making the change in variable $x = \sqrt{\Lambda\overline{\gamma}_b}$, we have

$$I = \lim_{m\to\infty} \frac{1}{2}\left\{1 - \frac{\Gamma\left(m+\frac{1}{2}\right)2x}{\Gamma(m)\sqrt{\pi m}} \right. \left. {}_2F_1\left(\frac{1}{2}; m+\frac{1}{2}; \frac{3}{2}; -\frac{x^2}{m}\right)\right\}. \quad (22)$$

By definition [11] the Gauss hypergeometric series is defined as

$${}_2F_1(a;b;c;x) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \cdot \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)} \frac{x^n}{n!} \quad (23)$$

and the series expansion of the error function as

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)n!}. \quad (24)$$

Using the identity of (23), (22) can be written as

$$I = \lim_{m\to\infty} \frac{1}{2}\left\{1 - \frac{1}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{\Gamma\left(n+\frac{1}{2}\right)}{\Gamma\left(n+\frac{3}{2}\right)} \cdot \right. \left. \frac{\Gamma\left(m+n+\frac{1}{2}\right) m^{-\left(n+\frac{1}{2}\right)}}{\Gamma(m)} \frac{(-1)^n x^{2n+1}}{n!}\right\}. \quad (25)$$

Making use of the recurrence formulas of the Gamma function [11], we can write

$$\Gamma\left(n+\frac{1}{2}\right) = \frac{1.3.5.7\ldots(2n-1)}{2^n}\Gamma\left(\frac{1}{2}\right) \quad (26)$$

and

$$\Gamma\left(n+\frac{3}{2}\right) = \frac{1.3.5.7\ldots(2n-1)(2n+1)}{2^n}\Gamma\left(\frac{3}{2}\right). \quad (27)$$

Therefore, using (26) and (27)

$$\frac{\Gamma\left(n+\frac{1}{2}\right)}{\Gamma\left(n+\frac{3}{2}\right)} = \frac{2}{(2n+1)}. \quad (28)$$

Further, let us consider

$$\lim_{m\to\infty} \frac{\Gamma\left(m+n+\frac{1}{2}\right) m^{-\left(n+\frac{1}{2}\right)}}{\Gamma(m)}. \quad (29)$$

Since $\Gamma(m)$ dominates $m^{\left(n+\frac{1}{2}\right)}$ and $\frac{\Gamma\left(m+n+\frac{1}{2}\right)}{\Gamma(m)} = 1$ as $m \to \infty$, (29) equals one.

Therefore, substituting (28) and (29) in (25), and making use of (24) we have

$$
\begin{aligned}
I &= \frac{1}{2}\left\{1 - \frac{1}{\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{2}{(2n+1)}\frac{(-1)^n x^{2n+1}}{n!}\right\} \quad (30)\\
&= \frac{1}{2}\{1 - \mathrm{erf}(x)\}\\
&= \frac{1}{2}\mathrm{erfc}(x).
\end{aligned}
$$

Substituting $x = \sqrt{\Lambda\overline{\gamma}_b}$ in (30) and recalling that

$$\frac{1}{\Lambda} = 1 + 2\frac{E_b}{N_0}\sigma_{ma}^2 \qquad (31)$$

and

$$\overline{\gamma}_b = \beta_1^2\frac{E_b}{N_0}, \qquad (32)$$

we have

$$P_e = \frac{1}{2}\mathrm{erfc}\left\{\left(\frac{N_0}{E_b} + 2\sigma_{ma}^2\right)^{-\frac{1}{2}}\right\} \qquad (33)$$

which is the same as the unfaded CPSK performance derived in [9].

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